# MATHEMATICAL MODELS FOR HEALTH SURVEY DATA AND ASSOCIATED MEASURES OF HEALTH STATUS*

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## I. INTRODUCTION

This paper presents the results of model building of measures of use and need of health services and measures of health status for a defined population. The data presented were obtained through a statewide household survey conducted by Rhode Island Health Services Research, Inc. (SEARCH) during the early part of 1972.** The sample was a full probability sample with households randomly selected from within each of Rhode Island's thirty-nine cities and towns. Interviews were obtained for 93 percent of the families falling in the sample. Information on disability and utilization of health services was obtained for 3,086 families consisting of 9,383 individuals. More detailed information describing the survey methodology is available in Thornberry et. al. (13).

Mathematical models to describe the state of health of a population are given in a paper by Chiang (4). In the same paper, he also proposes measures of health. In addition to Chiang's index of health, Miller (8) evaluated several other health indicies. Much of this theoretical work does not appear to have been applied in practice primarily due to a lack of appropriate data. This paper is concerned with validating the mathematical model proposed by Chiang and computing the indices of health using the survey data. In addition, techniques of the analysis of multi-dimensional contingency tables to describe the relationships between variables are applied to the data.

## II. MATHEMATICAL MODEL

In data analysis, mathematical models of phenomena are often developed for purposes of simplifying and summarizing raw data. Number of bed days per person per year (measuring need) and number of physician visits per person per year (use) are used in the study of mathematical models and in computing measures of health.

Chiang developed a model to predict the number of doctor's calls, clinic visits and the number of complaint periods that an individual had in a year. With N as the number of doctor's calls that an individual had in a year, Chiang has shown the probability distribution of N to be Poisson with expected value $\lambda$. Further, assuming $\lambda$ is a random variable with the gamma density function, the model proposed by Chiang for each of the variables that he selected is negative bionomial, i.e.,

$$P(N=n)=\frac{(n+\alpha-1)!}{(\alpha-1)!n!}\beta^{\alpha}(1+\beta)^{-(n+\alpha)},$$

$$n=0,1,2,...$$

Chiang fitted the negative bionomial model to the data on the number of doctor's calls and the number of complaint periods for different age groups and applied Chi-square goodness of fit test to validate the results. He suggested further testing of the model.

In the present study, the number of physician visits and the number of bed days are variables which closely correspond to variables chosen by Chiang. There were 9383 individuals responding to those two questions. The number of physician visits ranges from 0 to 90, while the number of bed days ranges from 0 to 365. Using the method of moments, the two parameters, $\alpha$ and $\beta$ are estimated. Expected frequencies are computed and are given in Tables 2.1 and 2.2.

It is apparent from Tables 2.1 and 2.2 that the observed and expected frequencies differ considerably and thus any goodness of fit test would reject the hypothesis of validity of the model. One might ask whether the model is appropriate or, if the model is appropriate, what might be the source for lack of fit. It is observed from the data that the distribution of N has several modes. For example, while the number of individuals with exactly 10 bed days was 175, only 38 and 23 individuals reported 9 and 11 bed days, respectively. Similarly, there were 141 people with 10 physician visits, while only 63 and 28 reported 9 and 11 physician visits, respectively.

The distribution of the number of bed days has peaks at 7, 10, 14, 20, 21, 28, 30 and troughs at other numbers. The individual's preference for these numbers is understandable for questions based primarily on memory recall. This phenomenon is also observed in another survey done by Brown University (2). In the SEARCH survey, the number of bed days within the last two weeks are also reported and this response is more likely to be accurate. More accurate response would be expected if an individual selected an interval for the number of bed

TABLE 2.1

Observed and Expected Number of Persons by the Number of Bed Days in a Year

| Bed Day | Observed | Expected |
|---|---|---|
| 0 | 4880 | 4135 |
| 1 | 505 | 1218 |
| 2 | 839 | 740 |
| 3 | 554 | 527 |
| 4 | 325 | 403 |
| 5 | 303 | 321 |
| 6 | 141 | 262 |
| 7 | 466 | 218 |
| 8 | 75 | 184 |
| 9 | 38 | 156 |
| 10 | 175 | 134 |
| 11 | 23 | 116 |
| 12 | 55 | 101 |
| 13 | 16 | 88 |
| 14 | 271 | 77 |
| 15 | 29 | 68 |
| 16 | 11 | 60 |
| 17 | 21 | 53 |
| 18 | 15 | 47 |
| 19 | 4 | 42 |
| 20 | 48 | 37 |
| 21 | 94 | 33 |
| 22 | 7 | 30 |
| 23 | 3 | 26 |
| 24 | 11 | 24 |
| 25 | 14 | 21 |
| 26 | 5 | 19 |
| 27 | 1 | 17 |
| 28 | 28 | 15 |
| 29+ | 359 | 14 |

SOURCE: SEARCH Household Survey

TABLE 2.2

Observed and Expected Number of Persons by the Number of Doctor's Visits in a Year

| Doctor's Visits | Observed | Expected |
|---|---|---|
| 0 | 2076 | 3129 |
| 1 | 2845 | 1632 |
| 2 | 1357 | 1094 |
| 3 | 726 | 789 |
| 4 | 603 | 588 |
| 5 | 287 | 447 |
| 6 | 326 | 344 |
| 7 | 108 | 268 |
| 8 | 129 | 210 |
| 9 | 63 | 165 |
| 10 | 141 | 130 |
| 11 | 28 | 103 |
| 12 | 302 | 82 |
| 13 | 20 | 66 |
| 14 | 17 | 52 |
| 15 | 59 | 42 |
| 16 | 16 | 34 |
| 17 | 12 | 27 |
| 18 | 13 | 22 |
| 19 | 3 | 17 |
| 20+ | 181 | 14 |

SOURCE: SEARCH Household Survey

TABLE 2.3

The Pooled Frequencies from Table 2.1

| Bed Day | Code | Observed | Expected |
|---|---|---|---|
| 0- 2 | 0 | 6224 | 5914 |
| 3- 5 | 1 | 1182 | 1395 |
| 6- 8 | 2 | 682 | 706 |
| 9-11 | 3 | 236 | 420 |
| 12-14 | 4 | 342 | 269 |
| 15-17 | 5 | 61 | 180 |
| 18-20 | 6 | 67 | 123 |
| 21-23 | 7 | 104 | 86 |
| 24-26 | 8 | 30 | 61 |
| 27- | 9 | 388 | 44 |

days such as between 9 and 11 or between 8 and 12 in a year. With this in mind, the data was grouped into the intervals 0-2, 3-5, 6-8, etc. Expected frequencies in the intervals were then computed using negative bionomial model. The difference in observed and expected frequencies in Table 2.3 are much smaller compared with those in Table 2.1. However, one may not conclude from this the validity of the negative bionomial model because of the arbitrariness involved in the pooling of the data. Still, if one were to argue in favor of the negative bionomial model in view of the good fit obtained by Chiang for the data that he collected, the expected frequencies in Tables 2.1 and 2.2 may be viewed as adjusted for bias. Due to cost limitations, no follow-up studies to determine the extent or bias of response preference for certain numbers were conducted. The effect of bias on the mean may cancel out thus not affecting the measures of health which are discussed later. It is interesting to note that in demographic studies, the digit preference or age heaping is observed in reporting of certain ages. The causes and patterns of age digit preference vary from one culture to another, but preference to report ages ending in '0' and '5' and avoid certain numbers as 13 and 4 is noted. Some techniques (Shryock and Siegel, (10)) are available to adjust the data for bias in census reporting but these have not been useful in the present context. Other contagious distributions such as Neyman's Type A are appealing because they may have more than one mode but the problem of bias must be resolved before another model is proposed.

Techniques developed by Goodman, Bishop, etc., to analyse the data of multi-dimensional contingency tables are used (Hanumara and Branson, (7)) to fit models. Using bed days as dependent (response) variable, age, sex, socio-economic characteristic as independent (factors) variables an unsaturated logit model is fitted. The results from the model are as follows: Main effects of economic status, age, sex on bed days are significant, interaction effects of economic status and age, age and sex on bed days are significant, while the interaction effect of economic status and sex on bed days is not significant. The factors economic status and sex affect the number of bed days independently within the age level.

## III. HEALTH STATUS INDEX

Traditionally death rate has been used as a crude health measure. However, in most industrialized countries, the death rate has been relatively stable over the past two decades. Thus measures of health incorporating not only mortality but also morbidity information are called for.

A health status index should fulfill two requirements: 1) it should be sensitive to significant changes in morbidity as well as mortality, 2) it should be subject to analysis into components providing a useful description of health problems underlying the index value. According to Goldsmith (6), a health status index serves three functions: Public information, administration, and medical science. Many of the recently proposed health indices have been evaluated by Miller (8). One of the criteria which Miller used for evaluation is the availability of input data. On a scale of 0 to 3, the index developed by Chiang (4) obtained a rating of 2, while the ratings for all other indices were 0 or 1. Chiang's index has a total rating of 10 out of a possible 13 in Miller's evaluation of thirteen indices. Miller's Q-index is the only other index which received a total rating of 10, while the rating for the rest ranged from 2 to 8. Miller's Q- index is much more program specific and the computation of this index is not feasible due to the unavailability of appropriate data. The interested reader will also find relevant discussions on health indices in Fanshel and Bush (5) and Sullivan (12).

### Chiang's Index

Chiang's index is based on the probability distribution of three variables; the frequency of illness; the duration of illness in number of days, and the time lost due to death in a year measured in number of days. He writes for age group x:

$$H_x = 1 - \bar{N}_x\bar{T}_x - (\tfrac{1}{2})M_x$$

where $H_x$ is the mean duration of health or the fraction of a year in which an individual is living and free of illness, $\bar{N}_x$ is the observed average number of illnesses per person, $\bar{T}_x$ is the average duration of an illness in a year, $M_x$ is the age-specific death rate for the year. The product $\bar{N}_x{*}\bar{T}_x$ is an estimate of the fraction of the expected duration that an individual is ill in a year. Now, H, the index of health for the entire population for the specified time period is defined to be

$$H = (I/P)\ \Sigma_x P_x H_x$$

where $P_x$ is the age-specific population and P is the total population. The value of H ranges from zero to one with healthier populations having larger values of H. In the extreme case, if no illness and no death for the population; i.e., $\bar{N}_x\bar{T}_x=0$, $M_x=0$ for each x; $H_x=1$ and $H=1$. If everyone were ill with no deaths for the entire year; i.e., $\bar{N}_x\bar{T}_x=1$, $M_x=0$, then $H_x=0$ and $H=0$.

### The Computation of Chiang's Index

In Chiang's model, illness is assumed to be a recognizable state of measurable duration, but how it is to be measured is not specified. The total duration of illness for an individual during a year has to be ascertained in order to compute Chiang's index. The number of bed days reported in the SEARCH survey data is only a part of illness period in a year for an individual. The total duration of illness for an individual is interpreted to be the sum of the number of bed days and the number of days with restricted activity but not in bed. Thus, the number of days of restricted activity apart from the days in bed had to be estimated. It should be noted that the term total duration of illness is consistent with the term "days of restricted activity" used in U. S. National Health Surveys. As the intent is to compute Chiang's index, his definitions are used here.

The number of bed days and the number of restricted activity days apart from the days in bed in a two week period are also available from the SEARCH survey data. Thus, (total duration of illness for two week period/bed days for two week period)*(bed days for twelve month period) is an estimate of the total duration of illness in a year. Data from two other surveys, Current Estimates (14) and Preliminary Report by the University of Chicago (15), are used to verify whether for specific-age group the ratios between total duration of

TABLE 3.1

Ratios from Current Estimates and SEARCH Household Survey

| | Age | | | |
|---|---|---|---|---|
| | Under 6 | 6-16 | 17-44 | 45- |
| Ratio of Restricted Days and Bed Days (Current Estimates, 12 months) | 2.358 | 2.083 | 2.249 | 2.409 |
| Ratio of Total Duration of Illness and Bed Days (SEARCH survey, 2 weeks) | 2.092 | 1.623 | 2.478 | 2.323 |

TABLE 3.2

Ratios (for 2 week period) from Preliminary Report and SEARCH Household Survey

| | Age | | | | |
|---|---|---|---|---|---|
| | 1-5 | 6-17 | 18-44 | 45-64 | 65- |
| Ratio of Restricted Days and Bed Days (Preliminary Report) | 1.671 | 1.565 | 1.955 | 1.947 | 1.767 |
| Ratio of Total Duration of Illness and Bed Days (SEARCH survey) | 2.220 | 1.660 | 2.478 | 2.382 | 2.248 |

TABLE 3.3

Statistics for R. I. 1971 from SEARCH Household Survey
Used to Compute Total Duration of Illness

| Age | Bed Days (2 weeks) | Restricted Days Exclusive of Bed Days (2 weeks) | Ratio of (2) + (3) and (2) | Bed Days (a year) |
|---|---|---|---|---|
| 0- 4 | 0.515 | 0.602 | 2.169 | 4.461 |
| 5-14 | 0.554 | 0.352 | 1.635 | 3.980 |
| 15-24 | 0.342 | 0.432 | 2.257 | 4.504 |
| 25-44 | 0.386 | 0.564 | 2.461 | 4.796 |
| 45-64 | 0.629 | 0.869 | 2.382 | 8.261 |
| 65- | 0.958 | 1.196 | 2.248 | 13.805 |

TABLE 3.4

Total Duration of Illness, Death Rate, and Population

| Age | Total Duration of Illness | Death Rate (1971) per 1000 | Population (1970) |
|---|---|---|---|
| 0- 4 | 9.676 | 4.10 | 76035 |
| 5-14 | 6.507 | 0.37 | 174163 |
| 15-24 | 10.166 | 0.89 | 173643 |
| 25-44 | 11.803 | 1.66 | 210297 |
| 45-64 | 19.678 | 10.30 | 208655 |
| 65- | 31.034 | 59.83 | 103932 |

illness and the number of bed days are in agreement with the ratio from the SEARCH survey data. This indicates the reliability of the ratio estimator, total duration of illness per person in a year.

The Current Estimates (14) gives restricted days and bed days for the year 1971. The ratios between them are computed and are shown in Table 3.1. This table also shows the ratios of total duration of illness and bed days for two week period, computed from SEARCH survey data.

In the Preliminary Report (15), the percentage of individuals who stayed in bed for one or more days during the last two weeks, and the average number of bed days experienced by those individuals who stayed in bed for one or more days during the last two weeks are given. The mean values of bed days of individuals can be computed by multiplying these two values. The mean values of restricted days can also be computed from this report. Then the ratios between bed days and restricted days are obtained. From SEARCH survey data, the mean values of bed days and restricted activity days not in bed as well as the ratios are computed. The results are shown in Table 3.2.

The National Health Surveys are carried out throughout the whole year and hence the estimates obtained from them are not influenced by the presence of an epidemic at a particular time of the year. There was an incidence of influenza at the time of SEARCH survey and this might have some small influence on the results relating to two week period. The difference in ratios in Table 3.1 are both positive and negative. On the other hand, the differences in ratios in Table 3.2 are all positive. It may be due to the fact that influenza mentioned before and also different geographic regions are involved.

For actual computation of the index, certain age groups are chosen for which the death rates are available. Table 3.4 gives the total duration of illness in a year as computed from the SEARCH survey data (Table 3.3) and the death rate for 1971 and the population for 1970 for the State of Rhode Island taken from Vital Statistics (9). Now, the health indices for different age groups are computed. For instance, the computations for age group zero to four are as follows:

$$H_x = 1 - \overline{N}_x\overline{T}_x - (\tfrac{1}{2})M_x$$

$$= 1-(9.676/365)-(\tfrac{1}{2})*.0041 = 0.971$$

| Age | 0-4 | 5-14 | 15-24 | 25-44 | 45-64 | 65- |
|---|---|---|---|---|---|---|
| $H_x$ | .971 | .982 | .972 | .967 | .941 | .885 |

Finally, Chiang's index H is obtained

$$H = (1/P)\Sigma_x P_x H_x$$

$$= 1/949725*(76035*0.971 + \ldots + 103932*0.885)$$

$$= 0.954$$

We have used certain age groups in computing H. The primary reason is the availability of data on death rates at these age groups. One can expect the value of H to be sensitive to different choices of age groups. Certainly, the result is improved with more age groups. However, it is not only the value of H that is interpreted but also individual values of $H_x$ representing age groups. Therefore, too many age groups are not needed and six age groups which are used here are enough for interpretation.

Recalling that population proportions used in computing H are for the year 1970 instead of 1971, one might question its effect on H. This is done because the data on population is available only up to 1970 at the present time and practically no change is expected in population proportions from 1970 to 1971.

Chiang gives formulas for sample variance of H but we are unable to compute it. In Chiang's terminology, total duration of illness is the product of number of illnesses and average duration of illness and the formula for sample variance of H requires data on these two variables. We interpreted the total duration of illness in a way convenient to compute H using SEARCH survey data but this does not permit the computation of the sample variance of H.

The only index which we are able to compute still has some limitations. We say the health status of Rhode Islanders in numerical terms is 0.954. As the larger the value of H, the healthier population is, one might conclude that Rhode Islanders are in fact healthy. It is hard to imagine a situation where the value of H will be close to zero and lead one to conclude otherwise. Hence no conclusion can be drawn using a single value. It would have been interesting to compare health index of 1971 with previous years but data are not available to permit such study. Again, for lack of necessary data, no comparisons of health status of states are made.

Health indices for different age

groups have some use. The reliability of H may be seen by different values of $H_x$. $H_x$ attains maximum value in the age group 5-14 and reaches the lowest value in the age group above 65 as one would expect. Health index might be used, for example, to study the health status of older people before and after medicare facilities.

At the time of conclusion of this study, Srivastava (11) is computing Chiang's index and also Chen's G- index (3) using different data base. Srivastava uses hospital days as a measure of illness as this is readily available for many sub-classifications of the population. Thus, health indices are being computed in his work for populations divided on the basis of census tracts or towns or social variables which lead to many comparative studies. Further, his studies show the G- index and Chiang's index are correlated.

FOOTNOTES

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